

Written Exam at the Department of Economics summer 2020

## Monetary Policy

Final Exam

10 June 2020

(3-hour open book exam)

Answers only in English.

***The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.***

**This exam question consists of 5 pages in total**

**This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.**

### **Be careful not to cheat at exams!**

You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispassing from the exam. In most cases, the student is also expelled from the university for one semester.

## PROBLEM A

Evaluate whether each of the following statements is true or false.  
Explain your answers carefully.

- 1) In the basic New Keynesian model featuring sticky prices, the welfare loss associated with inflation fluctuations is decreasing in the steady-state markup.
- 2) The uncovered interest parity (UIP) condition combined with the definition of the real exchange rate implies that the domestic nominal interest rate is given by the change in the real exchange rate plus domestic inflation. *Hint:* You may assume that the foreign nominal interest rate and foreign expected inflation are both zero.
- 3) In New Keynesian models with heterogeneous agents (so-called “HANK” models), intertemporal substitution of consumption plays a crucial role for the transmission mechanism of monetary policy.

## PROBLEM B

This problem asks you to solve a version of the New Keynesian model. Consider an economy described by the following three log-linear equations:

$$\pi_t = \beta \mathbf{E}_t \{\pi_{t+1}\} + \kappa \tilde{y}_t, \quad 0 < \beta < 1, \quad \kappa > 0, \quad (\text{B.1})$$

$$\tilde{y}_t = -\frac{1}{\sigma} (i_t - \mathbf{E}_t \{\pi_{t+1}\} - \rho) + \mathbf{E}_t \{\tilde{y}_{t+1}\}, \quad \sigma > 0, \quad \rho > 0, \quad (\text{B.2})$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t, \quad \phi_\pi > 1, \quad \phi_y > 0, \quad (\text{B.3})$$

where  $\pi_t$  denotes price inflation,  $\tilde{y}_t$  is the output gap, and  $i_t$  is the nominal interest rate. The monetary policy shock  $v_t$  evolves according to:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v, \quad 0 \leq \rho_v < 1,$$

where  $\varepsilon_t^v$  is an i.i.d. mean zero exogenous shock process. Finally,  $\mathbf{E}_t \{\cdot\}$  is the rational expectations operator conditional on all information up to and including period  $t$ .

1) Describe how equations B.1 and B.2 arise from optimal decisions by representative agents in the economy, and explain the assumption that  $\phi_\pi > 1$ .

2) Derive the solutions for  $\tilde{y}_t$  and  $\pi_t$  (*Hint*: conjecture that the solutions may be written as linear functions of the shock  $v_t$ ).

3) Use your solutions for  $\tilde{y}_t$  and  $\pi_t$  to show that the response of the nominal interest rate to a monetary policy shock can be written as:

$$i_t = \rho + \frac{(1 - \beta \rho_v)(1 - \rho_v)\sigma - \kappa \rho_v}{(1 - \beta \rho_v)[\phi_y + (1 - \rho_v)\sigma] + \kappa(\phi_\pi - \rho_v)} v_t. \quad (\text{B.4})$$

Is it possible to determine the sign of the response of the nominal interest rate to an increase in  $v_t$ ? Provide an explanation where you describe the different forces at play.

4) According to equation B.4, the slope of the New Keynesian Phillips Curve, as measured by  $\kappa$ , has a negative effect on the responsiveness of the nominal interest rate to a monetary policy shock. Explain.

## PROBLEM C

In this problem, you are asked to derive and discuss an optimal Taylor rule for monetary policy. Consider an economy described by the following three log-linear equations:

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \kappa x_t + u_t, \quad 0 < \beta < 1, \quad \kappa > 0, \quad (\text{C.1})$$

$$x_t = -\frac{1}{\sigma} (i_t - \mathbb{E}_t \{\pi_{t+1}\} - \rho) + \mathbb{E}_t \{x_{t+1}\} + \varepsilon_t, \quad \sigma > 0, \quad \rho > 0, \quad (\text{C.2})$$

$$i_t = \rho + \phi_\pi \pi_t, \quad \phi_\pi > 1, \quad (\text{C.3})$$

where  $\pi_t$  denotes price inflation,  $x_t$  is the output gap, and  $i_t$  is the nominal interest rate. The supply shock  $u_t$  and the demand shock  $\varepsilon_t$  are mutually uncorrelated i.i.d. mean zero exogenous shock processes with variance  $\sigma_u^2$  and  $\sigma_\varepsilon^2$ , respectively.

1) Solve the model for  $x_t$  and  $\pi_t$  (*Hint*: conjecture that the solutions may be written as linear functions of the shocks  $u_t$  and  $\varepsilon_t$ ), and show that the variances of  $x_t$  and  $\pi_t$  are:

$$\begin{aligned} \sigma_x^2 &= \left( \frac{\phi_\pi}{\sigma + \phi_\pi \kappa} \right)^2 \sigma_u^2 + \left( \frac{\sigma}{\sigma + \phi_\pi \kappa} \right)^2 \sigma_\varepsilon^2, \\ \sigma_\pi^2 &= \left( \frac{\sigma}{\sigma + \phi_\pi \kappa} \right)^2 \sigma_u^2 + \left( \frac{\kappa \sigma}{\sigma + \phi_\pi \kappa} \right)^2 \sigma_\varepsilon^2. \end{aligned}$$

2) Suppose that the social loss function of the central bank is given by:

$$\mathbb{L} = \eta \sigma_x^2 + \sigma_\pi^2, \quad \eta > 0. \quad (\text{C.4})$$

Show that the optimal value of the parameter  $\phi_\pi$  is given by:

$$\phi_\pi^* = \kappa \sigma \left( 1 + \frac{\kappa^2}{\eta} \right) \frac{\sigma_\varepsilon^2}{\sigma_u^2} + \frac{\kappa \sigma}{\eta}.$$

3) Explain in intuitive terms how the optimal value of  $\phi_\pi$  depends on the parameter  $\eta$  in the social loss function (C.4), and on the variances of each of the two shocks,  $\sigma_u^2$  and  $\sigma_\varepsilon^2$ .

4) Suppose there are no demand shocks, i.e., that  $\varepsilon_t = 0, \forall t$ , so  $\sigma_\varepsilon^2 = 0$ . Show that in this case, your solutions for  $x_t$  and  $\pi_t$  may be combined with the

solution for  $\phi_\pi^*$  to yield:

$$x_t = -\frac{\kappa}{\eta}\pi_t$$

Comment on this expression.